dependent on the level of bias present in the data, as expected. It may be emphasized that once the appropriate constants have been ascertained, no further trials are required, thus facilitating its use in real time processing of data.

Similarly, the other two observation residual-based models can also be effectively used in processing data. It was found that by using these models in various suitable sequential combinations, it is possible to significantly improve the resulting accuracy.

Finally, we present the results of simulation runs with noise model 4, which is based on vibration damping analogy. Table 3 shows filter runs with different initial errors in the position and velocity estimates. It is evident that the proposed filter is able to control the errors to within a much lower level rather quickly. With the filter progressing further in time, the error levels continue to fall, leading to an accurate satellite orbit estimation. It is interesting to note that this fast filter convergence is achieved in all cases regardless of the error levels in the initial guess.

The initial state covariance P_0 is not known a priori and has to be "guesstimated." In general, the filter performance is dependent on the accuracy of this guess as well. To assess the sensitivity of the noise model to the initial state covariance, the filter performance was evaluated for varying P_0. For filter runs, its base values were adopted from the corresponding elements of the observation noise covariance R. The results of this study with P_0 varied over several orders of magnitude are presented in Table 4. It can be seen that a rather rapid rate of convergence achieved by the filter remains virtually unaffected by the assumed values of the covariance P_0. It clearly establishes the importance of the proposed noise model in providing the self-adaptive filter characteristics and fast convergence, even under the most adverse situations. Thus, the mechanization of the last model appears to be not only independent of the initial guess of the state vector but also the state covariance. It is computationally more rugged as well as reliable and enables convergence in all cases tested. Furthermore, there being no constants here, unlike in the earlier three models, no tuning is required whatsoever.

Concluding Remarks

Presented here are four adaptive driving noise covariance models with a view to combat the filter divergence problems in implementation of the Kalman filter. The first three models are based on statistics of the observation residuals. These appear to make the filter quite robust, being effective regardless of the level of state errors in the initial guess. However, these require some trials to determine the proper noise participation constants. At times, reruns may have to be resorted to for achieving higher accuracies. However, the last noise model based on vibration damping analogy is fully robust. It appears to be totally insensitive to the initial state assumed. With no need for tuning the filter or even reruns, its versatility is clearly established.

References

[1]Bierman, G. J., and Thornton, C. L., "Numerical Comparison of Kalman Filter Algorithms: Orbit Determination Case Study," *Automatica,* Vol. 13, No. 1, 1977, pp. 23–35.

[2]Kumar, K., and Yadav, D., "Preliminary Orbit Determination Using Filtering Technquies: An Overview," *Proceedings of the IISc-ISRO Workshop-Cum Seminar on Preliminary Orbit Determination,* Jan. 1985, pp. 5.1.1–5.1.15.

[3]Anderson, B. D. O., and Moore, J. B., *Optimal Filtering,* edited by T. Kailath, Prentice-Hall, Englewood Cliffs, NJ, 1979.

[4]Morrison, N., *Introduction to Sequential Smoothing and Prediction,* McGraw-Hill, New York, 1969.

[5]Gelb, A. (ed.), "Optimal Linear Filtering," *Applied Optimal Estimation,* MIT Press, Cambridge, MA, 1974.

[6]Jazwinski, A. H., *Stochastic Process and Filtering Theory,* Academic, New York, 1970.

New Star Identification Technique for Attitude Control

B. V. Sheela,* Chandra Shekhar,*
P. Padmanabhan,† and M. G. Chandrasekhar‡
Indian Space Research Organization Satellite Centre, Bangalore, 560 017 India

I. Introduction

PRECISION pointing requirements are becoming one of the key features of many present day spacecraft. A star sensor, with[1] or without gyro,[2] is the answer to these stringent demands. For star sensor-based attitude determination, be it post facto or realtime, it is mandatory to identify the stars before attitude estimation can be carried out. In Ref. 3, Gottlieb has provided a comprehensive discussion of several star identification techniques: 1) direct match, 2) angular separation/phase match, and 3) discrete attitude variation.

The direct match technique, which associates a cataloged star with each observation by picking the closest star lying within a specified neighborhood of the observation transformed into the estimated celestial coordinates, requires that the initial attitude estimate be close to the true attitude. In the absence of a close initial estimate, the method requires accurate magnitude information for the imaged stars to resolve the problem of multiple candidate stars. By far, the angular separation technique, which matches the angular distances between pairs of imaged stars with those of cataloged stars, is the most widely used technique.[4] However, if the number of candidate stars is large, the method can run into ambiguities and misidentifications. The phase match technique is essentially a one-dimensional version of the angular separation technique and is suitable for spin-stabilized spacecraft. The discrete attitude variation technique, which is recommended as only a last resort by Gottlieb,[3] as it is essentially a method that repeatedly uses any of the techniques just mentioned for a host of attitude guesses, is very costly in terms of processing time. In this Note, yet another technique is presented that was the outcome of an

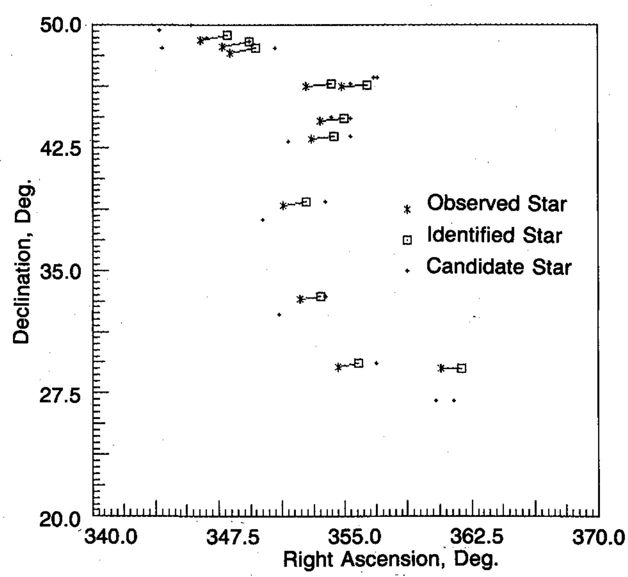

Fig. 1 Observed and candidate stars.

Received June 12, 1989; revision received Oct. 16, 1989. Copyright © 1990 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.

*Scientist, Flight Dynamics Division.

†Division Head, Flight Dynamics Division.

‡Scientific Secretary.

Table 1 **Inertial coordinates and direction cosines of observed and candidate stars**

Observed star number	Right ascension	Declination	Candidate star number	Right ascension	Declination	Angular shift	Slope parameters		
1	0.6690	28.98100	1	1.94900	29.03100	1.12054	−0.00087	0.01952	0.00076
			2	0.38500	27.03400	1.96298	0.01600	−0.00423	−0.03000
2	354.5880	29.10000	1	355.85501	29.30000	1.12390	−0.00009	0.01938	0.00305
3	352.2870	33.22500	1	351.07300	32.32300	1.36212	0.00585	−0.01886	−0.01324
			2	353.51901	33.43500	1.05056	0.00022	0.01808	0.00306
4	351.3100	38.94100	1	350.08301	38.12200	1.26171	0.00608	−0.01797	−0.01118
			2	352.67899	39.17600	1.08866	0.00002	0.01873	0.00318
5	351.2270	40.89000	1	349.46301	41.71200	1.55933	−0.01321	−0.02121	0.01078
			2	349.83200	42.01700	1.53720	−0.01585	−0.01586	0.01474
			3	351.64200	42.85000	1.98410	−0.02177	0.00873	0.02547
			4	353.51801	40.17500	1.88218	0.01207	0.02904	−0.00948
6	351.2090	40.88600	1	349.46301	41.71200	1.55003	−0.01322	−0.02097	0.01083
			2	349.83200	42.01700	1.53104	−0.01585	−0.01561	0.01479
			3	351.64200	42.85000	1.99018	−0.02178	0.00897	0.02553
			4	353.51801	40.17500	1.89336	0.01206	0.02928	−0.00943
7	351.1910	40.88300	1	349.46301	41.71200	1.54024	−0.01322	−0.02073	0.01087
			2	349.83200	42.01700	1.52423	−0.01585	−0.01537	0.01483
			3	351.64200	42.85000	1.99535	−0.02178	0.00921	0.02556
			4	353.51801	40.17500	1.90497	0.01206	0.02952	−0.00939
8	352.9640	43.05600	1	351.64200	42.85000	0.98924	0.00017	−0.01706	−0.00263
			2	354.24399	44.36800	1.60531	−0.01393	0.01781	0.01655
			3	354.39401	43.20600	1.05432	0.00023	0.01830	0.00191
			4	354.96100	44.27200	1.88810	−0.01192	0.02661	0.01535
9	353.5120	44.12400	1	351.64200	42.85000	1.86096	0.01211	−0.02546	−0.01613
			2	354.24399	44.36800	0.57835	−0.00198	0.00942	0.00305
			3	354.39401	43.20600	1.11792	0.01217	0.00991	−0.01159
			4	354.96100	44.27200	1.04930	0.00003	0.01822	0.00185
10	355.1000	45.06100	1	354.24399	44.36800	0.92209	0.00749	−0.01136	−0.00860
			2	354.24899	46.40200	1.46661	−0.01765	−0.00877	0.01634
			3	354.39401	43.20600	1.92285	0.02164	−0.01087	−0.02324
			4	354.96100	44.27200	0.79517	0.00949	−0.00256	−0.00979
			5	356.36700	46.35800	1.56992	−0.01501	0.01660	0.01581
11	354.8210	46.25600	1	354.24399	44.36800	1.93101	0.02264	−0.00928	−0.02317
			2	354.24899	46.40200	0.42111	−0.00249	−0.00669	0.00176
			3	354.96100	44.27200	1.98634	0.02465	−0.00048	−0.02437
			4	356.36700	46.35800	1.07279	0.00015	0.01868	0.00123
			5	356.74600	46.77100	1.42124	−0.00480	0.02354	0.00619
12	352.7050	46.24900	1	354.24899	46.40200	1.07712	0.00020	0.01871	0.00184
13	348.1860	48.28800	1	346.80701	49.23300	1.31119	−0.01554	−0.01280	0.01087
			2	348.00500	49.34400	1.06269	−0.01400	0.00083	0.01214
			3	349.30301	48.95400	0.99433	−0.00604	0.01434	0.00768
			4	349.74100	48.56300	1.06786	−0.00008	0.01836	0.00319
14	347.7510	48.66600	1	345.91400	49.98900	1.78411	−0.02181	−0.01636	0.01505
			2	346.80701	49.23300	0.84011	−0.00966	−0.00891	0.00650
			3	348.00500	49.34400	0.69817	−0.00812	0.00472	0.00776
			4	349.30301	48.95400	1.06185	−0.00016	0.01823	0.00331
			5	349.74100	48.56300	1.31959	0.00580	0.02226	−0.00119
15	346.4700	49.03900	1	343.98300	49.67300	1.73940	−0.01533	−0.02520	0.00721
			2	344.14200	48.62400	1.58754	−0.00151	−0.02725	−0.00477
			3	345.91400	49.98900	1.01625	−0.01375	−0.00311	0.01077
			4	346.80701	49.23300	0.29369	−0.00160	0.00434	0.00222
			5	348.00500	49.34400	1.04848	−0.00006	0.01797	0.00348
			6	349.30301	48.95400	1.86050	0.00790	0.03148	−0.00097

effort to identify stars without using the magnitude information for a post facto attitude determination task using a linear star sensor flown on the Indian Remote Sensing Satellite IRS-1A. Like the angular separation technique, this is also based on the assumption that the basic star pattern is conserved in the estimated celestial frame. The algorithm associated with this method uses the concept of a feature vector to identify an observation-candidate pair and has, therefore, the capability to resolve ambiguities that would result if only the angular distance measure is used for identification. In this Note, the algorithmic concept, typical results, a comparison of arithmetic operations involved, and the times on a microprocessor with those for the widely used angular separation method are presented.

II. Technique

This method, called the pattern match method, is based on the assumption that the pattern of the observations after transforming to the estimated coordinate frame wherein the attitude angles are assumed to be zero almost coincides with the

Table 2 Identification results

1st observed star is its	1st candidate star
2nd observed star is its	1st candidate star
3rd observed star is its	2nd candidate star
4th observed star is its	2nd candidate star
5th observed star is not identified	
6th observed star is not identified	
7th observed star is not identified	
8th observed star is its	3rd candidate star
9th observed star is its	4th candidate star
10th observed star is not identified	
11th observed star is its	4th candidate star
12th observed star is its	1st candidate star
13th observed star is its	4th candidate star
14th observed star is its	4th candidate star
15th observed star is its	5th candidate star

Table 3 Comparison of number of mathematical operations

Operation	Present algorithm	Angular separation method
Addition/ subtraction	4NO[a]*NS[b] + 4NO − 4	$2NOC_2 + 2(NS*NO)C_2$
Multiplication	4NO + 7NO*NS	$3NOC_2 + 3(NS*NO)C_2$
Division	4	—
Sine/cosine	(5NS + 4) NO	$4(NS+1)\ NO + NOC_2 + (NS*NO)C_2$
Square root	4	—

[a]NO = number of observations. [b]NS = number of candidates per observation.

Table 4 Comparison of times on NSSC-1

	Times	
	Present algorithm, s	Angular separation method, s
NO[a] = 6, NS[b] = 3	0.249	0.562
NO = 10, NS = 5	0.596	3.181

[a]NO = number of observations. [b]NS = number of candidates per observation.

pattern of the corresponding set of cataloged stars. As seen in Fig. 1, this means that the vectors that join observations to their catalog counterparts are all parallel to each other and have the same magnitude. The present algorithm exploits this property to identify the observations.

Let (O_i, $i=1$, NO) be the observed stars that have to be identified. The NC(i) candidate stars [C_{ij}, $j=1$, NC(i)] for each of the NO observed stars are obtained as the set of all stars in the catalog that are within a tolerance circle of, say, 2 deg from the estimated line of sight calculated with attitude angles assumed to be zero. A way of arriving at this set would be to partition the celestial sphere into a number of right ascension and declination cells. The stars in the cells that the 2-deg circle cuts are considered for extracting the candidate stars.

The line-of-sight (LOS) unit vector of an observed star O_i in the celestial frame, obtained by transforming the observed line and pixel number to the inertial frame assuming the attitude angles to be zero, is given by

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \begin{bmatrix} \cos\alpha_i & \cos\delta_i \\ \sin\alpha_i & \cos\delta_i \\ \sin\delta_i & \end{bmatrix} \tag{1}$$

Let the vectors of the candidate stars $C_{ij}, j=1$, NC(i) of O_i be given by

$$\begin{bmatrix} x_{ij} \\ y_{ij} \\ z_{ij} \end{bmatrix} = \begin{bmatrix} \cos\alpha_{ij} & \cos\delta_{ij} \\ \sin\alpha_{ij} & \cos\delta_{ij} \\ \sin\delta_{ij} & \end{bmatrix} \tag{2}$$

The equations for the lines joining O_i to C_{ij} are given by[5]

$$\frac{x-x_i}{x_{ij}-x_i} = \frac{y-y_i}{y_{ij}-y_i} = \frac{z-z_i}{z_{ij}-z_i} \qquad \begin{array}{l} i = 1 \cdots \mathrm{NO} \\ j = 1 \cdots \mathrm{NC}(i) \end{array} \tag{3}$$

The vector joining an observation O_{ik} to its candidate $C_{ik\,jl}$ is parallel and equal in magnitude to the vector joining O_{im} to $C_{im\,jn}$ if the slope parameters

$$P_{ik\,jl} - P_{ik} = P_{im\,jn} - P_{im} \quad \text{for} \quad P = x,y,z \tag{4}$$

Because of the constant magnitude and direction shift property described earlier, the feature vectors, comprising the three slope parameters $P_{ij} - P_i$ ($P = x,y,z$) and the angular shift parameter of the observations and their corresponding correct candidates, cluster around a single point in the four-dimensional feature space, and the feature vectors of the observations and their other candidate stars tend to fall away from this cluster. The identification is now a two-step procedure. In the first step, observation-candidate pairs are categorized into angular shift subintervals of, say, 0.2 deg with an overlap of 0.1 deg, resulting in about 21 subintervals. Then the subinterval that has the maximum number of observations that have a single candidate star, whose angular shift from the observation lies in this range, is found. Then the means and standard deviations of the slope parameters and the angular shift parameters of pairs in this subinterval are found. In the second step, an observation is identified as that candidate star for which the slope parameters lie within three or four σ limits of the mean. If no candidate exists in the limit, the observation is discarded as being unidentifiable.

III. Typical Results

For a typical set of observational data, Table 1 gives the inertial coordinates of observed and candidate stars, as well as the slope and angular shift parameters of all the observed-candidate pairs. Table 2 gives the final identification results. It is seen that some observations are not identified either because they correspond to spurious data or because stars with no spectral class have been deleted from the catalog used. Also seen is the observed star 13, which, had the angular distance measure alone been used as basis for identification, would have caused ambiguity. This star is unambiguously identified by the present algorithm. For this star, second and fourth candidate stars have close angular shift values, whereas the slope parameters fix the candidate star uniquely.

IV. Comparison of Processing Times on a Microprocessor

Since the algorithm is aimed at onboard application, the computation time for the algorithm is an important measure for evaluating it relative to other widely used methods like the angular separation method. The crux of the angular separation method lies in calculating the angular distances between all pairs of observed as well as candidate stars.[3] The number of arithmetic operations, such as addition, subtraction, multiplication, division, and sine/cosine operations, as a function of the number of observed stars NO and average number of candidate stars per observed star NS for this algorithm and the angular separation method, is given in Table 3. Table 4 gives the corresponding times (excluding other instructions) on the NASA Standard Spacecraft Computer (NSSC-1)[3,6] for both of the algorithms for various values of NO and NS. It can be seen that as NS increases the efficiency of the new algorithm is markedly better than that of angular separation method.

V. Conclusions

A new technique for star identification that does not use magnitude information was presented and its efficacy was demonstrated with typical results. A relative comparison of the estimated computational time on a microprocessor showed that the proposed algorithm offers a viable tool for onboard star identification.

Acknowledgments

The authors are grateful to N. Pant, Director, and K. Kasturirangan, Project Director, IRS-1A for their encouragement and support. Thanks are due to M. A. Shivaprakashan, S. Ramesh, S. Nirmala, and K. Madaiah for their help in preparing the manuscript.

References

[1]Dodds, S. J., "A Gyro Drift and Dynamics State Estimator for Satellite Attitude Control Systems Utilizing a Star Mapper," *International Journal of Control*, Vol. 39, No. 6, 1984, pp. 1275–1293.

[2]Gai, E., Daly, K., Harrison, J., and Lemos, L., "Star-Sensor-Based Satellite Attitude/Attitude Rate Estimator," *Journal of Guidance, Control, and Dynamics*, Vol. 8, No. 5, 1985, pp. 560–565.

[3]Wertz, J. R. (ed.), *Spacecraft Attitude Determination and Control*, Reidel, Dordrecht, Holland, 1978.

[4]Junkins, J. L., "Autonomous Star Sensing and Attitude Estimation," American Astronautical Society, Paper 79-013, 1979.

[5]Tuma, J. J., *Engineering Mathematics Handbook*, McGraw-Hill, New York, 1979.

[6]Dennehy, C. J., "Attitude Determination and Control Subsystem for the TOPEX Satellite," AIAA Paper 88-4129, 1988.

Book Announcements

HAIRER, E., LUBICH, C., and ROCHE, M., ***The Numerical Solution of Differential-Algebraic Systems by Runge-Kutta Methods,*** Lecture Notes in Mathematics, Vol. 1409, Springer-Verlag, Berlin, Heidelberg, 1989, 139 pages.

Purpose: This volume deals with numerical solution of differential equations with algebraic constraints and singular implicit differential equations, using the Runge-Kutta method.

Contents: Description of differential-algebraic problems; Runge-Kutta methods for differential-algebraic equations; convergence of index 1 problems; convergence of index 2 problems; order conditions of Runge-Kutta methods for index 2 systems; convergence for index 3 problems; solution of nonlinear systems by simplified Newton method; local error estimates; examples of differential-algebraic systems and their solutions.

BRENAN, K. E., CAMPBELL, S. L., and PETZOLD, L. R., ***Numerical Solution of Initial-Value Problems in Differential-Algebraic Equations,*** Elsevier Science Publishing Co., Inc., New York, 1989, 210 pages.

Purpose: This book deals with the mathematical structure of differential-algebraic equations (DAE's) and formulation and numerical methods applied to DAE systems.

Contents: Theory of DAE's; multistep methods; one-step methods; software for DAE's; applications.

MACIEJOWSKI, J. M., ***Multivariable Feedback Design,*** Addison-Wesley Publishing Co., Wokingham, England, U.K., 1989, 424 pages.

Purpose: This is a graduate text dealing with multivariable control design in the frequency domain.

Contents: Single-loop feedback design; poles, zeros, and stability; performance and robustness; multvariable design: Nyquist-like methods, LQG methods; Youla parameterization and H_∞ optimal control; design by parameter optimization; computer-aided design.

SUBRAHMANYAM, M. B., ***Optimal Control with a Worst-Case Performance Criterion and Applications,*** Lecture Notes in Control and Information Sciences, Vol. 145, Springer-Verlag, Berlin, Heidelberg, 1990, 133 pages.

Purpose: This volume deals with optimal control problems in which the cost functional is a product of powers of definite integrals. In particular, cost functionals of the form of a quotient of definite integrals and their relationships to finite-interval H_∞ control, performance robustness, and model reduction are treated.

Contents: Optimization with nonstandard cost functionals; linear control problems and an existence theorem; optimal disturbance rejection and performance robustness; necessary conditions for optimal disturbance rejection in linear systems; synthesis of finite-interval H_∞ controllers by state space methods; worst-case perfomance measures for linear control problems; model reduction with a finite-interval H_∞ criterion.

JUNKINS, J. L., ed., ***Mechanics and Control of Large Flexible Structures,*** Progress in Astronautics and Aeronautics, A. R. Seebass (editor-in-chief), Vol. 129, AIAA, Washington, DC, 1990, 705 pages.

Purpose: This book contains contributions from many authors on the subject. Collectively, the issues of high dimensionality, nonlinearity, and uncertainty are significant features of the 25 chapters of this book.

Contents: Structural modeling, identification, and dynamic analysis; control, stability, and optimization; control/structure interactions; analysis and experiments.

LUHANGA, M. L. and MWANDOSYA, M. J., ***Control System Analysis and Design Using the Smith Chart,*** Wiley Eastern Ltd., New Delhi, India, 1990, 106 pages.

Purpose: This book deals with the analysis and design of classical control systems using the Smith chart. The treatment is at the undergraduate level. The Smith chart is widely used by telecommunication engineers.

Contents: Frequency response plots on the Smith chart; stability; compensation; nonlinear system analysis using describing functions.